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ERROR ANALYSIS OF AN IMPROVED ALGORITHM FOR CALCULATING THE COORDINATES OF OBJECTS BASED ON AERIAL RECONNAISSANCE DATA FROM AN UNMANNED AERIAL VEHICLE

The calculation of geographical coordinates of objects based on aerial reconnaissance data from unmanned aerial vehicles in the face of enemy electronic warfare is an urgent task, the solution of which can increase the efficiency of the use of unmanned aerial vehicles for military missions. In particular, high-quality processing of aerial reconnaissance data obtained from unmanned aerial vehicles in the absence of signals from the GNSS global positioning system through the use of advanced algorithms and methods will help to improve the accuracy and efficiency of determining the coordinates of ground objects. The article proposes an improved algorithm that allows obtaining the coordinates of objects on flat terrain with high accuracy using aerial reconnaissance materials. To do this, the operator selects four points that are visible both on reconnaissance materials and on reference images that are georeferenced. The selected points allow to calculate the geographical coordinates of any point on the aerial reconnaissance materials. The accuracy of the coordinates will be higher for objects located inside a quadrangle with vertices at the selected points. The marked key points allow for a projective transformation that displays how the pixel coordinates of an object in an UAV image are transformed into its geographic coordinates. To ensure high accuracy of coordinate calculation, key points should be selected around the object, i.e., in such a way that it is located inside a quadrangle with vertices at the key points. As a result of the simulations, the maximum error inside the quadrangle of key points is less than 2 m, and the maximum error outside is about 17 m. The average error inside the quadrangle of key points was slightly more than 0.5 m, and outside - about 1 m. After the simulations, the improved algorithm was tested in field tests. For this purpose, several terrain areas were selected, and the coordinates of the objects located on them were determined. Several wing-type UAVs flew over these areas. The data obtained during the field tests do not differ much from those obtained during the simulations.

Keywords: *unmanned aerial vehicle complex; unmanned aerial vehicle; algorithm; computer vision; electronic warfare; projective transformation; error analysis.*

Problem statement in general. Considering the combat experience of using unmanned aerial vehicles (UAVs) in the context of armed aggression by the russian federation, it can be argued that the current method of determining the coordinates of ground objects in conditions of suppression of GNSS signals by electronic warfare (EW) means is not effective. In modern warfare between technologically advanced adversaries, electronic warfare (EW) is used intensively. Under conditions of jamming, payload operators are forced to manually determine the coordinates of ground objects (fixes) by comparing landmarks on third-party software products, which in turn reduces operational efficiency and has a negative impact on accuracy. Under such conditions, the task of calculating the geographical coordinates of objects located on aerial reconnaissance materials (ARM) received from unmanned aerial vehicles (UAVs) without the use of satellite global positioning systems becomes particularly relevant. One of the important tasks is to analyze the error of the calculated UAV's coordinates.

Accordingly, foreign and domestic scientists are faced with the task of improving software algorithms for processing UAV's location and determining the coordinates of objects on the Earth's surface with a certain accuracy.

Analysis of the latest research and publications. Calculating the coordinates of an object on the ARM from the UAV is a relevant issue that has been studied in a large number of scientific works. In particular, in [1], the authors focus on calculating the coordinates of an object in the coordinate system of the camera located on the UAV. Further, knowing the GPS coordinates and orientation of the UAV, as well as the position of the object relative to it, it is possible to obtain the absolute coordinates of the object on the Earth's surface that is in the camera's field of view. In [2], images from several UAVs are used to calculate the coordinates of an object. Its precise physical measurements can also be taken into account for localization, if known [3]. In addition, [4] proposes the implementation of an onboard smart information and computation system for autonomous navigation. The advantages of this study are the possibility of using non-identical imaging devices, but it does not provide for the possibility of determining coordinates and transmitting them to the operator of the target payload in real time. In publication [5], to calculate the coordinates of an object, it is tracked and images from a UAV taken from three different angles are used. In [6, 7], the YOLO detector is used to automatically search for an object in an image from the UAV, and the GPS coordinates of the drone and the position of the object in the image are taken into account to calculate its coordinates. An alternative method for determining the coordinates of an object is to use a UAV equipped with a stereo camera [8]. However, for high accuracy of the obtained coordinates, the object itself must be close to the stereo camera. In general, most studies use global positioning systems (GPS and similar systems) to calculate the geographical coordinates of objects on the UAV's image. In particular, the coordinates and orientation of the UAV are calculated using a global positioning system, a compass, and inertial measuring device. Knowing the camera rotation angles and pixel coordinates of the target on the image, it is possible to calculate the equation of the line on which the UAV and the object are located, as well as the intersection of this line with the Earth's surface. The disadvantage of the described method is the requirement to use a global positioning system, which is unreliable in modern combat conditions, particularly due to enemy jamming. GNSS signal spoofing can invisibly introduce errors into the GPS coordinates obtained by the UAV, even if an attempt is made to detect it by analyzing data from inertial measurement devices and global positioning system data on the UAV [9]. Another disadvantage of this method is the instability of the error in the calculated coordinates of objects relative to the error in determining the orientation of the UAV in space as the distance from the UAV to the object increases.

Formulation of the research task. The purpose of the article is to analyze the error of an improved algorithm for determining the coordinates of ground objects under the influence of enemy's EW means on UAVs in order to increase accuracy and efficiency in the absence of GNSS signals, although it is possible to use a video data transmission channel.

Core material. Consider the error in calculating the coordinates of a ground object if the position and orientation of the UAV are determined using GNSS, a compass, and inertial measurement device, and the coordinates of the object are calculated as the intersection with the Earth's surface of a straight line passing through the obtained coordinates of the UAV in the calculated direction.

Further, we assume that, when necessary, the following conditions are accepted for calculating the coordinates of an object based on ARM:

the surface of the Earth that falls on the ARM is completely flat;

the object is located on the Earth's surface;

the position of the UAV was calculated with zero error.

Under these assumptions, we determine the error in calculating the coordinates of the object as follows:

$$\Delta O = \sqrt{h^2 + d^2}\,\frac{\sin\beta}{\sin(\alpha-\beta)}, \qquad (1)$$

where h – the height of the UAV;

d – horizontal distance from the UAV to the object;

α – the angle between the Earth's surface and the line passing through the UAV and the ground object;

β – the error of calculating the orientation to the UAV, which led to the calculated line to the object passing in the same vertical plane as the line to the object and above it at an angle of β degrees.

That is, the error will be more than 110 m if $h = d = 1000$ m, $\alpha = 45°$, $\beta = 3°$. It will grow linearly in the event of an increase of h and/or d.

An example of determining the error in determining the coordinates of an object is shown in Fig. 1.

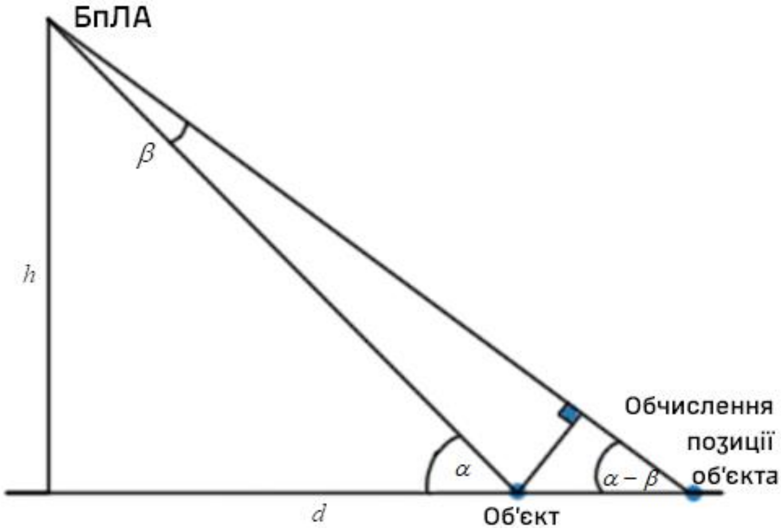

Fig. 1. Example of determining the error in determining the coordinates of an object

Let us assume that a rectangular coordinate system in metres (e.g. USK 2000 or WGS-84) is applied to the Earth's surface in a flat area above which the UAV is located, and that the height of points in space above the Earth's surface is used as the third coordinate. To increase the accuracy of the coordinates obtained, it is necessary to calibrate the camera so that the lines in the projection onto the matrix become straight lines. Further, we assume that the camera is already calibrated. The presence of a reference image linked to geographic coordinates means that we have a function P_S that maps each pixel coordinate $s=(s_1,s_2)$ in the reference images to the geographic coordinates of a specific point on the Earth's surface $c=(c_1,c_2)$.

Further, we will denote the image of the projective transformation (corresponding to the matrix A) of the point x as a linear transformation of a two-dimensional projective plane:

$$Ax=\left(\frac{a_{11}x_1+a_{12}x_2+a_{13}}{a_{31}x_1+a_{32}x_2+a_{33}},\frac{a_{21}x_1+a_{22}x_2+a_{23}}{a_{31}x_1+a_{32}x_2+a_{33}}\right), \tag{2}$$

where $A=(a_{ij})_{i=1,...,3,\,j=1,...,3}$ – matrix 3×3;

$x=(x_1,x_2)$ – two-dimensional coordinates.

The projection of points from the Earth's surface onto the ARM with UAV is a projective transformation from the plane on the Earth's surface to the plane of pixel coordinates on the ARM with UAV, i.e. there is a 3×3 matrix $Q=(q_{ij})_{i=1,...,3,\,j=1,..,3}$ such that

$$p=Qc, \tag{3}$$

where $c=(c_1,c_2)$ – geographical coordinates of a point on the Earth's surface;

$p=(p_1,p_2)$ – pixel coordinates of the corresponding point on the ARM from the UAV [2].

The inverse function to a two-dimensional projective transformation is also a two-dimensional projective transformation [2]. To define a two-dimensional projective transformation, it is sufficient to specify its values at four key points of the general position [2]. To do this, the operator can mark their correspondence, which is visible both on the ARM from the UAV and on the reference images (for example, road intersections, poles, corners of buildings, trees or bushes that stand out in some way), as shown in Fig. 2.

As shown, the top image is a photograph taken from a UAV, and the bottom image is a section of a satellite image that is georeferenced. By georeferencing the reference images, the geographical coordinates of the four marked points can be obtained.

Let $p^{(i)}=\left(p_1^{(i)},p_2^{(i)}\right)_{i=1,...,4}$ – the pixel coordinates of the four points marked on the ARM; $s^{(i)}=\left(s_1^{(i)},s_2^{(i)}\right)_{i=1,...,4}$ – the pixel coordinates of the corresponding marked points on the reference image, $Q=(q_{ij})_{i=1,...,3,\,j=1,..,3}$ – the projection transformation matrix from the plane of pixel coordinates on the reference image to the plane of their geographic coordinates. Let's determine

$c^{(i)} = \left(c_1^{(i)}, c_2^{(i)}\right)_{i=1,...,4}$ in this way: $c^{(i)} = Qs^{(i)}{}_{i=1,...,4}$. Then, using a direct linear transform, one can find [2] a projective transform that maps points $p^{(i)}{}_{i=1,...,4}$, y of point , so we get a matrix $R = \left(r_{ij}\right)_{i=1,...,3,\, j=1,...,3}$ which has $Rp^{(i)} = c^{(i)}{}_{i=1,..,4}$.

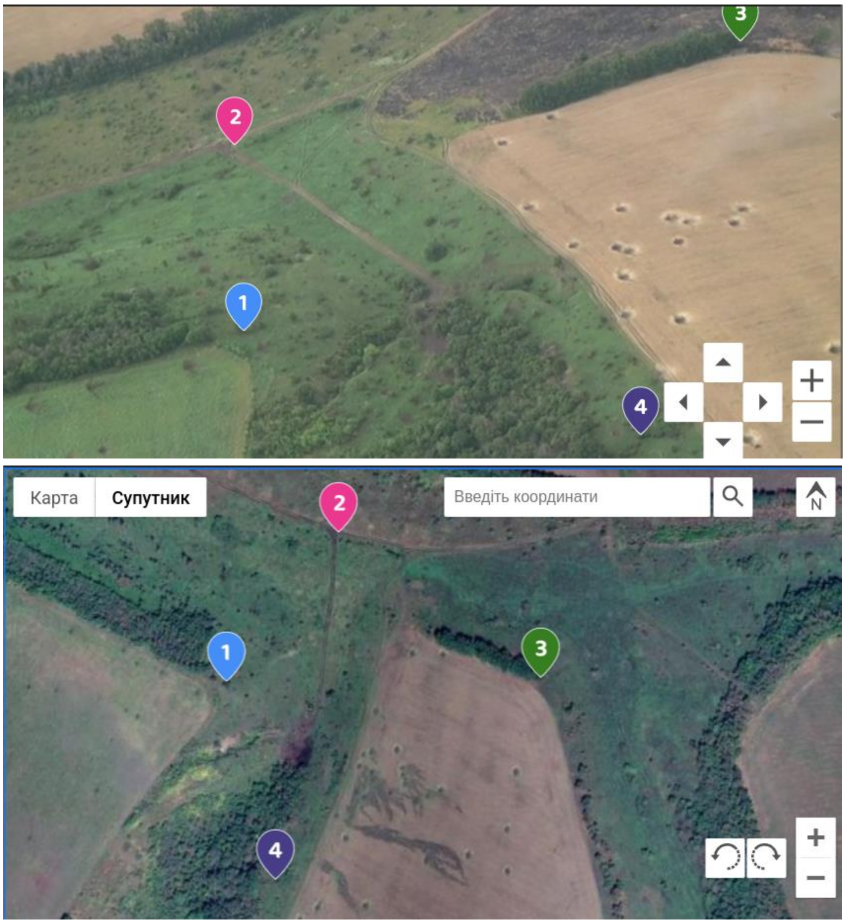

Fig. 2. Image from a UAV

To analyse the errors of the proposed method for calculating object coordinates, simulations of errors in calculating geographical coordinates were performed using a projective function corresponding to the matrix. R. The Earth's surface within the UAV's field of view was modelled as a flat plane. $z = 0$ in three-dimensional space, and the device itself is located at a point with random coordinates in the plane $z = 1000$, that is, at an altitude of 1000 m. In particular, let us have the following camera coordinates:

$$C = \begin{pmatrix} c_1 \\ c_2 \\ 1000 \end{pmatrix}, \quad (4)$$

where c_1, c_2 – random variables. We assume that the camera is directed at the origin, i.e. there is a point at the centre of its field of view.

$$V = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{5}$$

and the ‘up’ vector looks like this:

$$w = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \tag{6}$$

then the camera direction vector is

$$F = \frac{v - c}{\|v - c\|} = -\frac{c}{\|c\|}, \tag{7}$$

The vector ‘right’ for the camera can be calculated as a vector product

$$r = \frac{F \times w}{\|F \times w\|}, \tag{8}$$

and the vector ‘up’ also as a vector product

$$u = \frac{r \times F}{\|r \times F\|}. \tag{9}$$

Now the camera rotation matrix is

$$R = \begin{bmatrix} r_1 & r_2 & r_3 \\ -u_1 & -u_2 & -u_3 \\ f_1 & f_2 & f_3 \end{bmatrix}, \tag{10}$$

and the camera translation vector

$$t = -RC. \tag{11}$$

Let us assume that the optical centre of the camera is located in the centre of the image. Then we can obtain the internal matrix of the camera as follows:

$$K = \begin{pmatrix} f & 0 & w/2 \\ 0 & f & h/2 \\ 0 & 0 & 1 \end{pmatrix}, \tag{12}$$

where w – width of the image from the camera in pixels;

h – image height in pixels;

f – camera focal length.

For three-dimensional coordinates of a point $P = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ in global coordinates, we first move to the camera coordinates:

$$P_c = RP + t. \tag{13}$$

Next, the projection onto the image is performed using a projective transformation corresponding to the matrix K:

$$p = KP_c, \tag{14}$$

where p – pixel coordinates in an image.

Thus, we obtain a projective transformation H as inverted to the projection between the plane $z = 0$ and pixel coordinates of the image, where each point $w = \begin{pmatrix} x \\ y \end{pmatrix}$ transitions to its corresponding projection $p = \begin{pmatrix} p^{(1)} \\ p^{(2)} \end{pmatrix}$ in pixel coordinates on the image as follows:

$$p = K\left(R\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} + t \right), \tag{15}$$

where K – internal camera matrix;

t – camera translation vector.

So, $\begin{pmatrix} x \\ y \end{pmatrix} = Hp$.

Let's determine four random points $(p_i)_{i=1,\ldots,4}$ in pixel coordinates of the image, and their corresponding projections onto the plane of geographic coordinates $w_i = Hp_i, i = 1,\ldots,4$. Pixel coordinates $(p_i)_{i=1,\ldots,4}$ we will interpret as modelling the pixel coordinates of objects in the image that are visible both in the UAV photograph and in the satellite image (reference image), $w_i = (Hp_i)_{i=1,\ldots,4}$ – their exact geographical coordinates. Since in practice the marking of corresponding points on the image and their projection onto the plane of geographical coordinates is carried out with an error, we will model the error using a random vector:

$$w_i = w_i + \begin{pmatrix} \delta_{i,x} \\ \delta_{i,y} \end{pmatrix}, \tag{16}$$

where $\delta_{i,x}$ та $\delta_{i,y}$ – independent, identically distributed, random variables with uniform distribution in the interval $[-1,1]$.

Then, taking into account the error in practice, we obtain a projective transformation corresponding to the matrix H, in particular $H p_i = (w_i)_{i=1,\ldots,4}$. For each pexil p on the image we will find the error $e(p) = \|H p - Hp\|$.

Let S_{in} – is a a set of pixels inside a rectangle with vertices at points $(p_i)_{i=1,\ldots,4}$, and S_{out} – a set of points located outside the same quadrilateral.

Denote the average error inside and outside the specified rectangle, respectively, E_{in} та E_{out}, so:

$$E_{in} = \frac{1}{|S_{in}|} \sum_{p \in S_{in}} e(p), \; E_{out} = \frac{1}{|S_{out}|} \sum_{p \in S_{out}} e(p). \tag{17}$$

Similarly, we will determine the maximum errors:

$$E_{in}^{\max} = \max_{p \in S_{in}} e(p), \; E_{out}^{\max} = \max_{p \in S_{out}} e(p), \tag{18}$$

where $E_{in}^{\max}$ – maximum error within a rectangle with vertices at points $(p_i)_{i=1,...,4}$;

$E_{out}^{\max}$ – maximum error outside this same rectangle;

$e(\cdot)$ – error fuction.

So, for each of the 10 randomly generated $(p_i)_{i=1,...,4}$ 1,000 simulations of random errors were performed $\delta_{i,x}$ and $\delta_{i,y}$. The Python programming language was used for this purpose. The results are presented in Table 1.

Table 1

Simulations results

UAV coordinates (camera_position)	Maximum error within a rectangle from key points	Maximum error outside the rectangle from key points	Average error within a rectangle from key points	Average error outside the rectangle from key points
(25, -101, 1000)	1,61	3,71	0,55	0,77
(103, 6, 1000)	1,63	3,79	0,55	0,77
(-235, -441, 1000)	1,68	13,40	0,56	0,96
(-101, 486, 1000)	1,74	11,73	0,57	0,94
(-133, -387, 1000)	1,58	8,58	0,56	0,88
(213, -16, 1000)	1,71	4,96	0,57	0,82
(310, 93, 1000)	1,51	6,77	0,56	0,86
(31, -148, 1000)	1,62	4,19	0,56	0,77
(-272, 51, 1000)	1,63	6,09	0,56	0,84
(-454, 229, 1000)	1,60	17,02	0,56	1,01

As a result of simulations, normalised errors of pixel projection into geographical coordinates were visualised in Fig. 3. In particular, the sides of a rectangle with vertices at points $(p_i)_{i=1,...,4}$ shown in white. The size of the error is indicated by a grey scale: from the lightest (smallest error) to black as the error increases.

As a result of 10 series of 1,000 simulations, the maximum error inside the quadrangle from key points was less than 2 m, and the maximum error outside was about 17 m. The average error inside the quadrangle from key points was slightly more than 0.5 m, and outside it was about 1 m. The smallest average errors are located inside the convex hull of key points. Outside the convex hull of key points, the error increases as the distance to the convex hull increases.

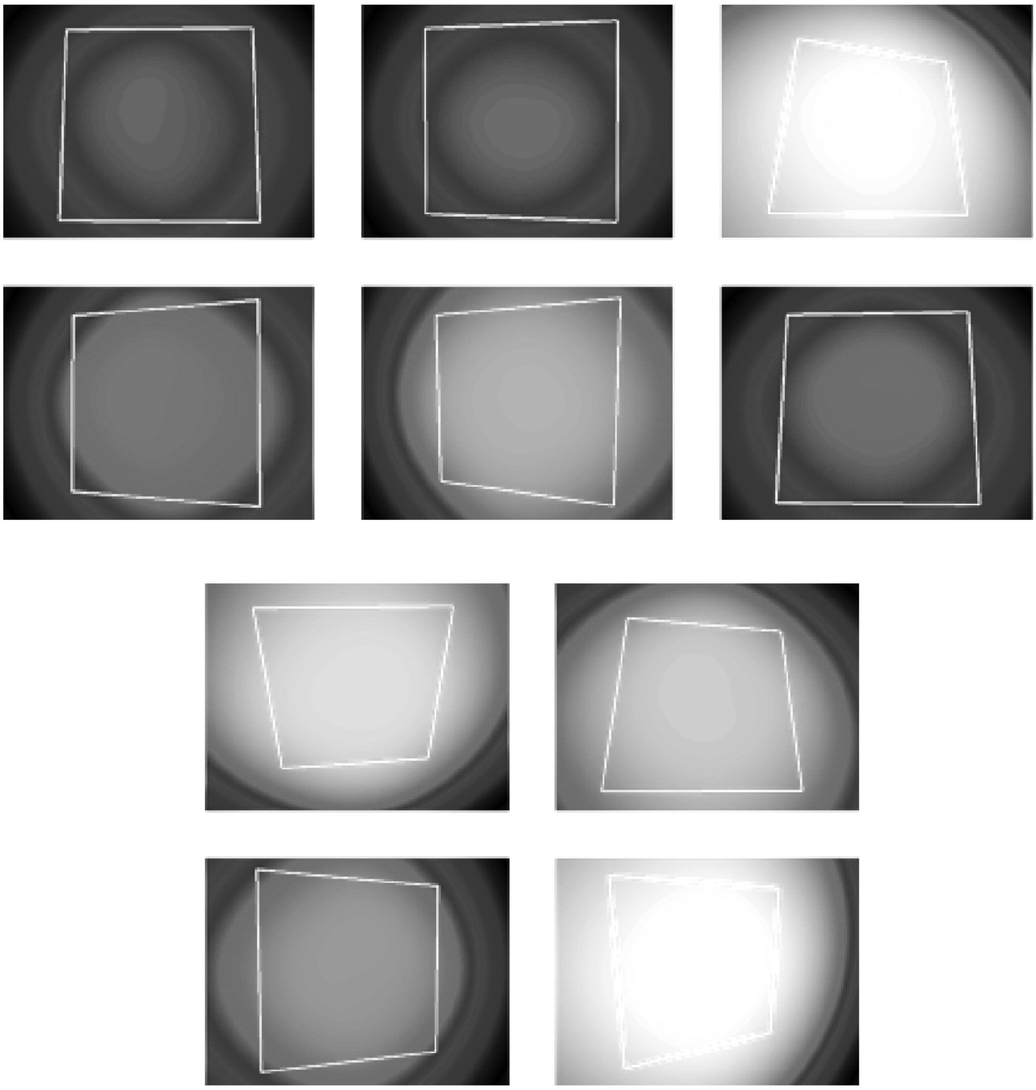

Fig. 3. Distribution of normalised pixel projection errors

After conducting simulations, the improved algorithm was tested in field trials. For this purpose, several areas of terrain were selected and the coordinates of the objects located on them were determined. These areas were flown over by several wing-type UAVs. A video stream was received from the ground station in near real time and processed on a test computer running software based on the improved algorithm. During the flights, about 800 control measurements were made. The maximum error inside the quadrangle was about 4 m, which was greatly influenced by factors such as cloud cover and convective air currents that distorted the images in the video stream. Outside the quadrangle, in the near zone, the error was about 8 m. Also, at a considerable distance outside the quadrangle, when the improved algorithm was still processing the image, the maximum error was about 28 m. The results of field tests do not differ significantly (correlate) from the data obtained during simulations. (practically confirm the results obtained during simulation)

Conclusions. The error of the improved algorithm for calculating point coordinates based on aerial reconnaissance data from a UAV flying over flat terrain has been verified. To calculate the coordinates, the operator marks four points on the ARM from the UAV and the corresponding points on the reference image, which is linked to geographical coordinates. The marked key points allow calculating the projective transformation that maps the pixel coordinates of the point on the ARM from the UAV to the geographic coordinates of this point. For high accuracy in calculating the coordinates of an object, key points must be selected around it, i.e. in such a way that it is located inside a quadrangle with vertices at the key points.

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